Spatiotemporal scaling for out-of-equilibrium relaxation dynamics of an elastic manifold in random media: Crossover between diffusive and glassy regimes

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We study the relaxation dynamics of a three-dimensional elastic manifold in a random potential from a uniform initial condition by numerically solving the Langevin equation. We observe the growth of the roughness of the system up to larger wavelengths with time. We analyze the structure factor in detail and find a compact scaling ansatz describing two distinct time regimes and the crossover between them. We find that a short-time regime corresponding to a length scale smaller than the Larkin length L_c is well described by the Larkin model, which predicts a power-law growth of the domain size $L(t)$. Longer-time behavior exhibits a glassy regime with slower growth of $L(t)$.

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I. INTRODUCTION

Fluctuations around macroscopically condensed states such as charge density waves[1] (CDWs) and flux-line lattices in superconductors[2] often exhibit glassy dynamics due to frustration between the elastic restoring forces originating from the stiffness of the ordered state and random pinning forces brought on by impurities. A thermally activated process dominates the slow relaxation dynamics of such systems such as spin glasses (see, e.g., Refs. 3–6) and supercooled liquids.[7]

An important basic problem in the studies of glassy dynamics is isothermal aging—i.e., relaxation at a fixed temperature from initial states far from equilibrium.[5] The aging of elastic manifolds in random media has been studied theoretically by dynamical mean-field theories[8] and numerical simulations.[9–15] Presumably there exists a dynamical length scale $L(t)$ which grows with time t such that the system is equilibrated on wavelengths smaller than $L(t)$.[15–17] In other words, $L(t)$ is the size of the local equilibrium domain. The roles of $L(t)$ have been examined extensively in the context of the aging of spin glasses (see, for instance, Refs. 18–20). While $L(t)$ grows algebraically without random pinning forces, frustration drastically slows it down. Typically one expects that the growth law becomes logarithmic due to the energy barriers which grow with the length scale.[11,15,20–24] The purpose of the present paper is to analyze the aging of an elastic manifold in random media in terms of $L(t)$ and investigate the crossovers between the two characteristic regimes: the so-called Larkin regime and glassy regime.

A standard theoretical model to study the above-mentioned CDW-*like* systems is the elastic manifold model in a random potential—e.g., the Fukuyama-Lee-Rice model,[25,26] given by the Hamiltonian

$$H = \int d\mathbf{r}\left[\frac{1}{2}\kappa|\nabla\theta(\mathbf{r},t)|^2 - h(\mathbf{r})\cos[\theta(\mathbf{r},t)-\beta(\mathbf{r})]\right]. \quad (1)$$

Physically the scalar field $\theta(\mathbf{r})$ at position $\mathbf{r}$ in space is understood as the local fluctuation of the phase part of the order parameter of the condensate, such as the CDW state. The first term with elastic constant κ indicates the elastic deformation energy which is minimized when $\theta(\mathbf{r},t)$ is spatially uniform in the absence of the second term, the random field energy. This sinusoidal random potential is a periodic function with respect to $\theta(\mathbf{r},t)$, reflecting the underlying periodicity of the condensate. Both the amplitude $h(\mathbf{r})$ and phase $\beta(\mathbf{r})$ of the random field are quenched random variables with short-ranged spatial correlations. Hereafter $\langle\cdots\rangle$ means a thermal average and $\overline{\cdots}$ means an average over the quenched randomness (samples).

Let us recall here some basic *static* properties of the system which is understood better than the dynamical properties of interest. It is believed that the physical properties of this kind of systems, such as the roughness characterized by $B(\mathbf{r})=\overline{\langle[\theta(\mathbf{r})-\theta(\mathbf{0})]^2\rangle}$, are different on three distinct length scales.[27] First, in the very-short-length regime, perturbative analysis of the effects of disorder predicts algebraic growth of the roughness with distance r, $B(\mathbf{r})\propto r^{2\zeta}$, with some roughness exponent ζ. Below four dimensions it is known that $\zeta=(4-d)/2$.[25,28,29] Then the perturbative regime, which we call the *Larkin regime* in the following, must be terminated at the so-called Larkin length L_c over which the effect of randomness overcomes the elasticity. Then the so-called *random manifold regime* begins[30–33] where many metastable states exist and the roughness of the system is characterized by a nontrivial roughness exponent ζ_{rm}. In much larger length scales, the amplitude of θ eventually grows beyond the period of the random potential. If the periodicity is relevant, the system cannot gain more benefit from the potential energy at a cost of elastic energy. Then the last regime, called the Bragg glass regime, begins. In three dimensions the large wavelength fluctuation is highly suppressed and $B(\mathbf{r})$ is no longer expressed by the algebraic functions $r^{2\zeta}$, but by a certain logarithmic function of the distance r.[27,34,35] Then it is said that the system is in the Bragg glass phase where the system has quasi-long-range order (QLRO). In the case of a single-harmonic potential as the present model, Eq. (1), the end of the Larkin regime and the beginning of the Bragg

glass regime coincide, i.e., the transient random manifold regime does not exist.[27,36] This is because the potential has only a single characteristic scale, that is a period 2π, and does not have another smaller scale which yields the upper bound of the Larkin regime such as the short-ranged correlation length of potential along the θ direction.

In this paper, we study the out-of-equilibrium relaxation dynamics of the elastic system in the periodic random potential, Eq. (1). This system shows different types of dynamics at different time scales. We show that each of these is related to the equilibrium spatial property in the corresponding length scale. Although we discuss in particular the case of a three-dimensional system, we may comment on systems in general d dimensions.

In the next section, we review the dynamics in the Larkin regime, which can be examined analytically. In Sec. III, a numerical analysis of the structure factor is given. In the Sec. IV, we propose a scaling law which describes the crossover from the Larkin regime to the glassy regime. In the final section, we present conclusions and remarks.

II. POWER-LAW DOMAIN GROWTH IN THE LARKIN REGIME

When the phase fluctuation is very small, $\theta(\mathbf{r},t) \ll 1$, the Hamiltonian, Eq. (1) is reduced to the so-called Larkin model,[28] which is exactly solvable. When the second term on the right-hand side (rhs) of Eq. (1) is expanded by θ up to linear order, the disorder effect is represented by the quenched random force $\eta(\mathbf{r}) = -h(\mathbf{r})\sin\beta(\mathbf{r})$, which is supposed to be a random Gaussian number satisfying

$$\overline{\eta(\mathbf{r})} = 0, \quad \overline{\eta(\mathbf{r})\eta(\mathbf{r}')} = \Delta_h \delta(\mathbf{r}-\mathbf{r}'), \tag{2}$$

with a finite variance Δ_h. The overdamped Langevin equation is written as

$$\gamma \frac{d}{dt}\theta(\mathbf{r},t) = -\frac{\delta H}{\delta \theta(\mathbf{r},t)} + \zeta(\mathbf{r},t) = \kappa \nabla^2 \theta(\mathbf{r},t) + \eta(\mathbf{r}) + \zeta(\mathbf{r},t), \tag{3}$$

where γ is the friction coefficient and $\zeta(\mathbf{r},t)$ is a Gaussian white thermal noise satisfying

$$\langle \zeta(\mathbf{r},t)\rangle = 0, \quad \langle \zeta(\mathbf{r},t)\zeta(\mathbf{r}',t')\rangle = 2\gamma T\, \delta(\mathbf{r}-\mathbf{r}')\delta(t-t'), \tag{4}$$

T being the temperature of the heat bath. The average $\langle \cdots \rangle$ is taken over independent noise realizations.

The formal solution with the uniform initial condition $\theta(\mathbf{r},0)=0$, for all $\mathbf{r}$, is expressed as

$$\theta(\mathbf{q},t) = \frac{1}{\gamma}\int_0^t dt'\ e^{-\kappa q^2 (t-t')/\gamma}[\eta(\mathbf{q}) + \zeta(\mathbf{q},t')], \tag{5}$$

where the functions of $\mathbf{q}$ are the Fourier transformations of the corresponding functions of $\mathbf{r}$. The structure factor—i.e., the Fourier transform of the scalar correlation function—is obtained as

$$B(\mathbf{q},t) \equiv \int d\mathbf{r} \overline{\langle \theta(\mathbf{0},t)\theta(\mathbf{r},t)\rangle} e^{i\mathbf{q}\cdot\mathbf{r}} = \overline{\langle |\theta(\mathbf{q},t)|^2\rangle}$$
$$= T\frac{1-e^{-2q^2L(t)^2}}{\kappa q^2} + \Delta_h \left[\frac{1-e^{-q^2L(t)^2}}{\kappa q^2}\right]^2, \tag{6}$$

where

$$L(t) \equiv \sqrt{\kappa t/\gamma}. \tag{7}$$

Equation (6) means that the two fluctuations owing to temperature and randomness are decoupled in the Larkin model. But the growth of the two is characterized by a single time-dependent length $L(t)$ such that the system is equilibrated over a wavelength shorter than $L(t)$—i.e., $B(\mathbf{q},t) \sim B(\mathbf{q},t \to \infty) = T/\kappa q^2 + \Delta_h/\kappa^2 q^4$ for $qL(t) \gg 1$.

From Eq. (6), we can evaluate the amplitude of phase fluctuation as

$$\sigma(t) \equiv \overline{\langle \theta(\mathbf{r},t)^2\rangle} = \int d\mathbf{q}\ B(\mathbf{q},t) \tag{8}$$

$$\approx C_0 - \frac{C_1}{d-2}\frac{T}{\kappa}L(t)^{-(d-2)} + \frac{C_2}{4-d}\frac{\Delta_h}{\kappa^2}L(t)^{4-d}, \tag{9}$$

where C_1 and C_2 are positive constants. The third term diverges in equilibrium ($t \to \infty$, $L(t) \to \infty$) below four dimensions. By writing $\sigma(t) \propto L(t)^{2\zeta}$ one can read off the roughness exponent of the Larkin model as $\zeta_{\text{Larkin}} = (4-d)/2$. In the long-length (-time) scale, $L(t) \gg L_T \equiv \sqrt{\kappa T/\Delta_h}$, the third term due to the quenched randomness is dominant and the second term due to the thermal fluctuation term can be ignored.

Note that the growth law of $L(t)$ given by Eq. (7) is the same as in the absence of the random potential—i.e., diffusive dynamics with dynamical exponent $z=2$. It means that the quenched random potential does not bring pinning effects at the level of its linear approximation. It is easy to see that this is the case at any higher levels of perturbative treatments of the random potential.

III. GLASSY DYNAMICS IN THE NONLINEAR POTENTIAL

The linear approximation adopted in the previous section breaks down for large $\sigma(t)$, which occurs when $L(t)$ becomes as large as the so-called Larkin length L_c:

$$L_c(\Delta_h) = \left(\frac{C_2}{c_L^2 \kappa^2}\Delta_h\right)^{-1/(4-d)}. \tag{10}$$

This length is derived from the threshold condition $\sigma(t) = (2\pi c_L)^2$, where c_L is a constant, similar to Lindemann's constant,[37] indicating the border below which the nonlinearity of the random potential can be ignored. The corresponding time scale is

$$t_c(\Delta_h) = L_c(\Delta_h)^2 \gamma/\kappa \propto \Delta_h^{-2/(4-d)}. \tag{11}$$

Beyond the Larkin length the nonlinearity of the random potential yields many metastable states. The potential barri-

ers between them will significantly slow down the growth of $\sigma(t)$. Hereafter we call this nonperturbative regime just the *glassy regime* which may correspond to the random manifold or Bragg glass regime. Now we study the growth of roughness in the glassy regime by numerical simulations.

A. Simulations

In practice we consider the lattice version of Eq. (3). The equation of motion for the phase at the lattice point $\mathbf{r}_i$ is

$$\Gamma \frac{d}{dt}\theta_i(t) = -J \sum_{j\in \text{n.n.}} [\theta_i(t) - \theta_j(t)] - h_i \sin(\theta_i(t) - \beta_i) + \zeta_i(t), \tag{12}$$

where

$$\langle \zeta_i(t)\zeta_j(t')\rangle = 2\Gamma T \delta_{ij}\delta(t-t'). \tag{13}$$

Hereafter we set the coupling constant J and the friction coefficient Γ to unity.

Here let us explain some details of the numerical simulations. Phase variables $\theta_i(t)$'s are put on the cubic lattice with size $N=L_{\text{sys}}^3=128^3$ and periodic boundary conditions are imposed in all directions. The phases β_i's are introduced as independent uniform random numbers between 0 and 2π. On the other hand, the strength of the random field h_i is set to a uniform value h for all sites,[42] so that

$$\Delta_h = h^2/2. \tag{14}$$

We investigate the relaxation dynamics at various values of the random field $h=1.0-5.0$ and temperatures $T=0.0$, 0.5, 1.0, and 1.5.[43] We numerically solve Eq. (12) by the second-order stochastic Runge-Kutta method.[38] In the initial state, θ_i is set to 0 for all i. Physical quantities are averaged over 128 runs at least, each of which has independent realizations of the random phase β_i and thermal noise $\zeta_i(t)$.

We define the lattice version of the structure factor, Eq. (6), as

$$B(\mathbf{q},t) = N\overline{\langle |\theta(\mathbf{q},t)|^2\rangle}, \tag{15}$$

with $\theta(\mathbf{q},t)=N^{-1}\Sigma_j \theta_j(t)e^{i\mathbf{q}\cdot\mathbf{r}_j}$ (Ref. 44).

B. Results

Figure 1 shows some examples of the profile of $B(q,t)$. Components for all q are zero at $t=0$, and the structure factor grows with time. It can be seen that at larger q the amplitude saturates to be a t-independent but q-dependent value while at smaller q the amplitude remains t dependent but q independent. The above observation suggests that there is indeed a dynamical length scale $L(t)$ which grows with time t such that components satisfying $qL(t)\gg 1$ become equilibrated: the system has become rough on short wavelengths, but remains flat at larger wavelengths.

A simplest scaling which connects the dynamical regime $qL(t)\ll 1$ and the static regime $qL(t)\gg 1$ may be[1012,15,39]

$$B(q,t) = q^{-(d+2\zeta)}\tilde{B}(qL(t)), \tag{16}$$

where ζ is the roughness exponent and the scaling function behaves as $\tilde{B}(x)\sim$const for $x\gg 1$ and $\tilde{B}(x)\sim x^{d+2\zeta}$. By integrating over q one obtains the corresponding scaling form for

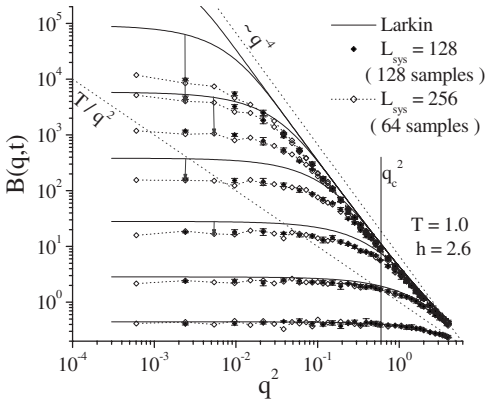

FIG. 1. Snapshots of the profile of the structure factor $B(q,t)$. The time changes uniformly in logarithmic scale as $t/0.34 = 4^0, 4^1, \ldots, 4^6$ from the bottom to the top. The symbols with error bars are data for $L_{\text{sys}}=128$, and dotted lines are for $L_{\text{sys}}=256$. The arrows connect the values in the Larkin model (bold curves) given by Eq. (6) and numerical data obtained in the sinusoidal model at the same time t. The vertical line shows $q^2=t_c^{-1}$ (Ref. 47).

$$\sigma(t) \equiv \sum_{\mathbf{q}\neq 0} B(\mathbf{q},t) = \overline{\langle \theta(\mathbf{r},t)^2\rangle} \tag{17}$$

as

$$\sigma(t) \propto L(t)^{2\zeta}. \tag{18}$$

[Note that the above scaling holds only if the system size L_{sys} is sufficiently larger than $L(t)$ for a given time t.] Indeed one can find easily that the Larkin model discussed in Sec. II satisfies these scalings exactly at $T=0$. However, the real behavior will be more complicated even in the Larkin regime because roughness originates not only from the quenched random field, but also from the thermal noise at finite temperatures. Furthermore, there will be a crossover from the Larkin regime to the glassy regime at L_c. In Sec. IV we perform a more elaborate scaling analysis taking into account these complications.

The analytic solution of the Larkin model is also plotted in Fig. 1 for comparison. In a very short time the structure factors of the two models coincide. As time goes by, it becomes apparent that $B(q,t)$ for the sinusoidal potential model, Eq. (12), does not grow as fast as that of the Larkin model, Eq. (3). As shown later, the time scale beyond which the equivalence breaks down is t_c given by Eq. (11). Furthermore, by a closer inspection it appears that the envelope function $B(q,t\to\infty)$—i.e., the equilibrium structure factor—changes from that of the Larkin model. The structure factor for small-q parts seems to be slightly different from q^{-4} of the Larkin model. We regard these changes as the crossover from the Larkin regime to the glassy regime, which we analyze more carefully in Sec. IV.

Figure 2 shows the time evolution of $\sigma(t)$. In the Larkin regime $L(t)\ll L_c$ or $t\ll t_c$, we expect $\sigma(t)\propto L(t)^{2\zeta}\sim t^{2\zeta/z}$ [see Eq. (18)] with $\zeta=1/2$ and $z=2$. However, the data deviate from this behavior for $t>t_c$. The crossover time increases as

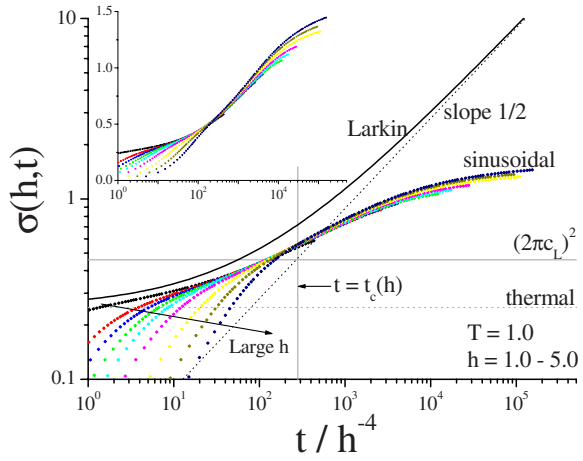

FIG. 2. (Color online) Time evolution of $\sigma(t)$. The inset is a semilogarithmic plot of the same data. The solid line curves show the results of the Larkin model for $h \ll T$. The horizontal dotted line indicates the equilibrium value $\sigma(t=\infty)$ for $h=0$. The lines denoted with $2\pi c_L$ and $t_c(h)$ indicate the crossover (see the details in Ref. 47).

the strength of the random field h decreases. We consider that this reflects a crossover from the Larkin regime to the glassy regime. Indeed, by simply scaling t by the anticipated crossover time $t_c \propto h^{-4}$ given in Eq. (11), the data of $\sigma(t)$ collapse onto a universal function for sufficiently large t. The growth of $\sigma(t)$ for $t > t_c$ is very slow presumably due to the activated glassy dynamics.

IV. SCALING OF THE STRUCTURE FACTOR

As observed in the previous section, the crossover from the Larkin regime to the glassy regime can be described by a simple scaling in which the time t is scaled by the crossover time $t_c(\Delta_h)$ given by Eq. (11) corresponding to the Larkin length $L_c(\Delta_h)$. Now we analyze the spatiotemporal scaling law of the structure factor $B(q,t)$ itself, which provides us more detailed information than the integrated one σ in Eq. (17). The basic idea is expressed by Eq. (16) which connects the dynamic $qL(t) \ll 1$ and static $qL(t) \gg 1$ regimes. However, we need to take into account complications due to the roughness of different origins: (i) thermal roughness with the roughness exponent $\zeta_{\text{thermal}}=(2-d)/2$, (ii) roughness due to the random field in the Larkin regime with $\zeta_{\text{Larkin}}=(4-d)/2$, and (iii) roughness in the glassy regime which has a different q dependence from (ii).

A. Scaling ansatz

We propose the following scaling ansatz. Most importantly the crossover from the Larkin to the glassy regime is taken into account by scaling the length (or the wave number) by the Larkin length L_c given by Eq. (10) and the time by the corresponding time scale t_c given by Eq. (11). We propose that the structure factor takes the form

$$B(q,t) = TL_c^2 \frac{1 - e^{-2Y(qL_c)X(t/L_c^2)}}{Y(qL_c)} + \Delta_h L_c^4 \left[\frac{1 - e^{-Y(qL_c)X(t/L_c^2)}}{Y(qL_c)} \right]^2. \tag{19}$$

This is an extended version of Eq. (6): the first and second terms describe the fluctuation due to thermal noise[45] and quenched randomness, respectively. The dynamical and static regimes described in the simplest scaling, Eq. (16), correspond to $XY \ll 1$ and $XY \gg 1$, respectively: in the dynamical regime $XY \ll 1$, the structure factor is t dependent but q independent, while in the static regime $XY \gg 1$, it becomes t independent but q dependent.

We suppose that the scaling functions X and Y take the following asymptotic forms in the Larkin regime:

$$X(\tilde{t}) = \begin{cases} \tilde{t} & \text{for } \tilde{t} \ll 1, \\ X_g(\tilde{t}) & \text{for } \tilde{t} \gg 1, \end{cases} \tag{20}$$

$$Y(\tilde{q}) = \begin{cases} \tilde{q}^2 & \text{for } \tilde{q} \gg 1, \\ \text{const} \times \tilde{q}^{d/2+\zeta_g} & \text{for } \tilde{q} \ll 1. \end{cases} \tag{21}$$

The scaling function $Y(\tilde{q})$ with $\tilde{q}=qL_c$ describes the equilibrium structure factor: $Y(\tilde{q})^{-2}=\tilde{q}^{-4}$ in the Larkin regime $\tilde{q} \gg 1$ and $\tilde{q}^{-(d+2\zeta_g)}$ in the glassy regime $\tilde{q} \ll 1$. The exponent ζ_g is an unknown roughness exponent in the glassy regime which will be smaller than ζ_{Larkin}. Particularly ζ_g will be zero if the system has quasi-long-range order. On the other hand, the scaling function $X(\tilde{t})$ with $\tilde{t}=t/t_c$ describes the growth law of $L(t)$ in the Larkin regime $\tilde{t} \ll 1$ and the glassy regime $\tilde{t} \gg 1$. More precisely, the dynamical length $L(t)$ can be estimated by solving

$$Y(L(t)^{-1}L_c)X(t/L_c^2) = 1. \tag{22}$$

The function $X_g(\tilde{t})$ is an unknown increasing function of the scaled time $\tilde{t}$ in the glassy regime which will be slower than any algebraic functions due to the anticipated activated dynamics.[11,15]

For $\tilde{t} \ll 1$ and $\tilde{q} \gg 1$, the above scaling reproduces Eq. (6) in the Larkin regime. However, it turns out that certain vertex corrections are needed in the coefficients as

$$T \to T(1 + \alpha_{T\Delta}\Delta_h + \cdots) \tag{23}$$

and

$$\Delta_h \to \Delta_h(1 - \alpha_{\Delta T}T + \cdots) \tag{24}$$

in analyzing the raw data. These coefficients appear when performing a perturbation expansion of the random potential beyond the linear approximation in Sec. II. In the following analysis we treated the first-order correction terms only and regarded them as fitting parameters.

B. Numerical analysis

Now let us examine the validity of the scaling law presented above using our numerical data. Scaling functions $X(\tilde{t})$ and $Y(\tilde{q})$ are determined by least-squares fitting.

First we perform the fitting by fixing the temperature T. For example, Fig. 3 shows the result of the scaling for T

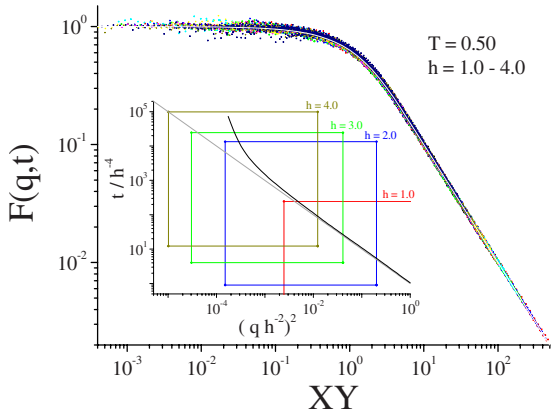

FIG. 3. (Color online) Result of the scaling at $T=0.5$. $F(q,t)$ is plotted as a function of $X(t/t_c)Y(qL_c)$ for $h=1.0$, 1.6, 2.0, 2.5, 2.7, 3.0, 3.2, 3.5, and 4.0. The white curve indicates $(1-e^{-XY})/XY$. Here the coefficients are corrected with $\alpha_{T\Delta}=0.027$ as in Eq. (23) and $\alpha_{\Delta T}=0.22$ as in Eq. (24). The rectangles in the inset show the data ranges of our simulations in t/t_c and $q^2L_c^2$ for $h=1.0$, 2.0, 3.0, and 4.0. The bold curve in the inset indicates $X(t/t_c)Y(qL_c)=1$. The space-time region above this curve is equilibrated.

$=0.5$. We plotted $F(q,t)$ as a function of XY where

$$F(q,t)=\frac{1}{X}\sqrt{\frac{B(q,t)}{L_c^4\Delta_h}-\frac{T}{L_c^2\Delta_h}\frac{1-e^{-YX}}{Y}},$$

which leads to $(1-e^{-XY})/XY$ if the scaling law in Eq. (19) is valid. The figure shows a nice collapsing of data using the proper scaling functions X and Y.

Next let us bring together data at different temperatures. To this end we note that the Larkin length can weakly depend on temperature:

$$L_c(\Delta_h,T)=[\Delta_h/c_L(T)^2]^{-1/(4-d)}. \tag{25}$$

The constant $c_L(T)$ will be larger for higher temperatures because thermal fluctuations will mask the quenched random potential over short length scales. In practice we treated $c_L(T)$ as a fitting parameter. Then, as shown in Fig. 4, the scaling function Y for different temperatures can be laid on a universal curve independent of temperature. The resultant correction factor $c_L(T)$ is shown in the inset of Fig. 4.

The scaling function $X(\tilde{t})$ exhibits a crossover from power-law domain growth to glassy dynamics. Looking at Fig. 4 carefully, the scaling function X is not universal, which is more apparent at lower temperature. Particularly the relaxation stops on the way at zero temperature. This indicates that the relaxation in the long-time regime is a thermally activated process. The system cannot escape from a metastable state without thermal assistance.

The scaling function Y^{-2} represents the structure factor in the equilibrium state $(X\to\infty)$. From the present scaling we obtain its shape even for small wave numbers $\tilde{q}\ll 1$ where $B(q,t)$ is still far from equilibrium. While the growth law $X(t)$ turned out to depend on the temperature T, we find that $Y(t)$ is essentially independent of the temperature T. This means that the spatial correlation function in equilibrium has a universal form independent of both T and Δ_h. The long-

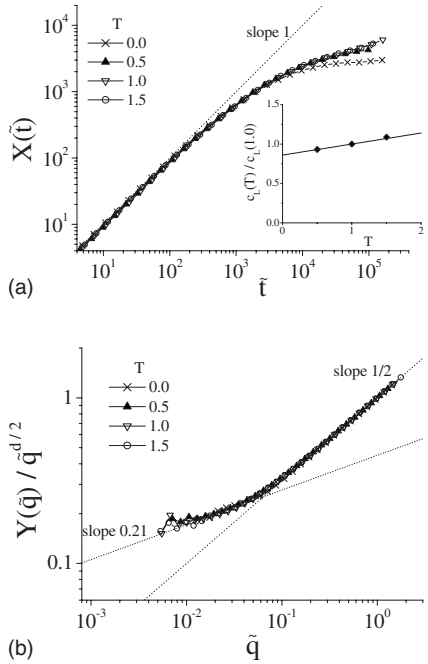

FIG. 4. Scaling function $X(\tilde{t})$ and $Y(\tilde{q})$ for several fixed temperatures. The inset in the top graph indicates the correction factor $c_L(T)$. Here we use $c_L(T=0)$ by extrapolating from finite-temperature data. More precisely, the horizontal axes in this plot, $\tilde{t}$ and $\tilde{q}$, are scaled values using $L_c(\Delta_h,T)=[hc_L(1.0)/c_L(T)]^{-2}$ instead of that in Eq. (25).

wavelength behavior of Y seems to obey a power law $Y(\tilde{q})\sim\tilde{q}^{-(d/2+\zeta)}$. The roughness exponent ζ is smaller than that in the Larkin regime, and the value is consistent with the one for random manifolds, $\zeta_{\rm rm}\approx 0.21$ (Ref. 46). However, it is more natural to expect that this agreement is a transient behavior that ζ is approaching zero because the present single-harmonic model is considered to take a single crossover to the Bragg glass regime.[27,36]

V. SUMMARY AND DISCUSSION

In this paper the relaxation dynamics of the three-dimensional elastic manifold in a random potential has been studied. We especially focused on the crossover between the Larkin regime and the glassy regime—i.e., power-law domain growth and thermally activated relaxation. We proposed a scaling method for the dynamical structure factor which encodes the dynamical growth of the roughness of different origins and successfully applied it to explain the crossover between the Larkin and glassy regimes. At a given temperature the out-of-equilibrium structure factor can be scaled by using the dynamical length $L(t)$ and the Larkin length $L_c(\Delta_h)$. Quite interestingly our analysis yields the structure factor in equilibrium, which is hard to observe in equilibrium simulations. The temperature dependence can be also taken care of only by introducing a correction factor $c_L(T)$ for the Larkin length. It turns out that the scaling function $Y(\tilde{q})$ is universal and independent of temperature.

Although we analyzed a model with a random potential that is periodic with respect to the deformation field, an in-

dication of QLRO was hardly observed. This will appear when the amplitude of the phase fluctuation becomes greater than the period of the random potential, $\sigma(t) \gg (2\pi)^2$. The crossover region between the Larkin and Bragg glass regimes, however, persists for quite a long time and has special importance in the dynamical aspect. This is because glassy behavior becomes serious at the early stage of the crossover. From the obtained scaling function $Y(\tilde{q})$, we can roughly estimate the end of the Larkin regimes as $c_L = \sqrt{\sigma(t_c)}/2\pi \approx 0.1$ in Eq. (10).[47] The growth rate of $\sigma(t)$ decreases quite quickly before $\sigma(t)$ reaches $(2\pi)^2$ (see Fig. 2). (In fact, almost all of $|\theta_i|$ in our simulations are smaller than π and the system does not *feel* that the potential is periodic.) Therefore the early stage of the crossover, which has a sufficiently long time range, hardly reflects the periodicity of the random potential and is similar to the crossover to the random manifold regime.

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[42] We consider that the distribution of the amplitude h_i on each site does not change the semiquantitative property in the weak-pinning regime because the system feels an averaged random potential over the region where the phase is almost uniform. We checked it by preliminary simulations.
[43] These temperatures are lower than the ferromagnetic transition temperature, ~2.20, of the pure XY model on the cubic lattice, whose spin-wave approximation is the present model.
[44] More precisely we calculated $B(q_0,t)=[B(\mathbf{q}_0=(q_0,0,0),t)+B(\mathbf{q}_0=(0,q_0,0),t)+B(\mathbf{q}_0=(0,0,q_0),t)]/3$ where $q_0=2\pi n/L_{\mathrm{sys}}$ for $n=0,1,2,\ldots,L/2$. Hereafter we use $q^2=2(1-\cos q_0)=q_0^2+O(q_0^4)$ instead of q_0^2. By using this q, the same formula with that of continuum model, such as Eq. (6), can be used for the Larkin model on the lattice.
[45] Strictly speaking the first term in Eq. (19), the structure factor of purely thermal origin, does not need to be scaled in the same

way as the second term. (One can argue that purely thermal roughness exists only in the range $qL_c \gg 1$.) This is an artifact of our scaling form, which is chosen for simplicity, but does not make significant changes in the analysis in the small-q regimes of our interest where the second term is dominant.

[46] The roughness exponent for the d-dimensional system with an N-component deformation field is written as $\zeta_{\mathrm{rm}}(d,N)=(4-d)/[4+\nu(d,N)N]$ (Ref. 40). The exponent $\nu(3,1)$ corresponding to the present system is not known, but expected to be between $\nu(1,1)=1/2$ (Ref. 20) and $\nu(4,1)=0.80$ (Ref. 41). Substituting these ν's and $d=3$ to the above formula instead of $\nu(3,1)$ leads to close values of ζ_{rm}, 0.22 and 0.21, respectively.

[47] From Fig. 4 we can read the crossover wave number $\tilde{q}=q_c h^2 \approx 0.06$ and then $t_c=1/q_c^2 \approx 280/h^4$ and $L_c=t_c^{1/2} \approx 17/h^2$. As a result $c_L=(1/2\pi)[C_2\Delta_h L_c/\kappa^2]^{1/2} \approx 0.11$ where $C_2 \approx 0.056$, $\Delta_h = h^2/2$, and $\kappa=1$, corresponding to $J=1$.